

NAG Toolbox for MATLAB

s17ac

1 Purpose

s17ac returns the value of the Bessel Function $Y_0(x)$, via the function name.

2 Syntax

```
[result, ifail] = s17ac(x)
```

3 Description

s17ac evaluates an approximation to the Bessel Function of the second kind $Y_0(x)$.

Note: $Y_0(x)$ is undefined for $x \leq 0$ and the function will fail for such arguments.

The function is based on four Chebyshev expansions:

For $0 < x \leq 8$,

$$Y_0(x) = \frac{2}{\pi} \ln x \sum_{r=0}^l a_r T_r(t) + \sum_{r=0}^l b_r T_r(t), \text{ with } t = 2\left(\frac{x}{8}\right)^2 - 1.$$

For $x > 8$,

$$Y_0(x) = \sqrt{\frac{2}{\pi x}} \left\{ P_0(x) \sin\left(x - \frac{\pi}{4}\right) + Q_0(x) \cos\left(x - \frac{\pi}{4}\right) \right\}$$

where $P_0(x) = \sum_{r=0}' c_r T_r(t)$,

and $Q_0(x) = \frac{8}{x} \sum_{r=0}' d_r T_r(t)$, with $t = 2\left(\frac{8}{x}\right)^2 - 1$.

For x near zero, $Y_0(x) \simeq \frac{2}{\pi} \left(\ln\left(\frac{x}{2}\right) + \gamma \right)$, where γ denotes Euler's constant. This approximation is used when x is sufficiently small for the result to be correct to **machine precision**.

For very large x , it becomes impossible to provide results with any reasonable accuracy (see Section 7), hence the function fails. Such arguments contain insufficient information to determine the phase of oscillation of $Y_0(x)$; only the amplitude, $\sqrt{\frac{2}{\pi x}}$, can be determined and this is returned on soft failure. The range for which this occurs is roughly related to **machine precision**; the function will fail if $x \gtrsim 1/\text{machine precision}$.

4 References

Abramowitz M and Stegun I A 1972 *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Clenshaw C W 1962 Mathematical tables *Chebyshev Series for Mathematical Functions* HMSO

5 Parameters

5.1 Compulsory Input Parameters

1: **x – double scalar**

The argument x of the function.

Constraint: $x > 0.0$.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **result – double scalar**

The result of the function.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

x is too large. On soft failure the function returns the amplitude of the Y_0 oscillation, $\sqrt{2/(\pi x)}$.

ifail = 2

$x \leq 0.0$, Y_0 is undefined. On soft failure the function returns zero.

7 Accuracy

Let δ be the relative error in the argument and E be the absolute error in the result. (Since $Y_0(x)$ oscillates about zero, absolute error and not relative error is significant, except for very small x .)

If δ is somewhat larger than the machine representation error (e.g., if δ is due to data errors etc.), then E and δ are approximately related by

$$E \simeq |xY_1(x)|\delta$$

(provided E is also within machine bounds). Figure 1 displays the behaviour of the amplification factor $|xY_1(x)|$.

However, if δ is of the same order as the machine representation errors, then rounding errors could make E slightly larger than the above relation predicts.

For very small x , the errors are essentially independent of δ and the function should provide relative accuracy bounded by the *machine precision*.

For very large x , the above relation ceases to apply. In this region, $Y_0(x) \simeq \sqrt{\frac{2}{\pi x}} \sin\left(x - \frac{\pi}{4}\right)$. The amplitude $\sqrt{\frac{2}{\pi x}}$ can be calculated with reasonable accuracy for all x , but $\sin\left(x - \frac{\pi}{4}\right)$ cannot. If $x - \frac{\pi}{4}$ is written as $2N\pi + \theta$ where N is an integer and $0 \leq \theta < 2\pi$, then $\sin\left(x - \frac{\pi}{4}\right)$ is determined by θ only. If $x \gtrsim \delta^{-1}$, θ cannot be determined with any accuracy at all. Thus if x is greater than, or of the order of the inverse of *machine precision*, it is impossible to calculate the phase of $Y_0(x)$ and the function must fail.

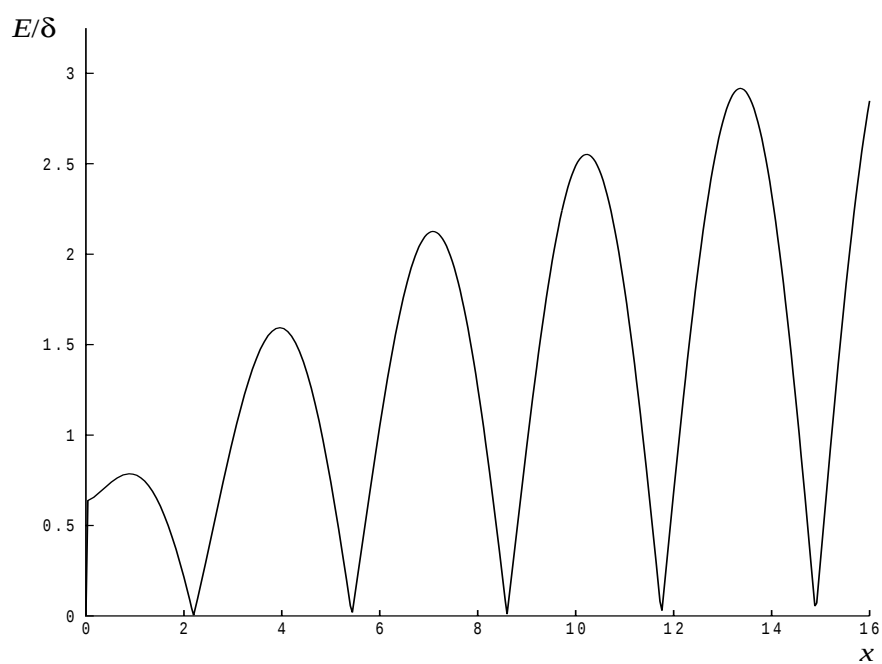


Figure 1

8 Further Comments

None.

9 Example

```
x = 1;  
[result, ifail] = s17ac(x)
```

```
result =  
    0.0883  
ifail =  
    0
```